CONTROL OF A NONSTATIONARY PROCESS OF THERMOELECTRIC COOLING BY VARIATION OF THE GEOMETRIC SHAPE OF THE BRANCHES OF A THERMOELEMENT

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We show the possibility of controlling transient processes at the operating junction of a thermoelement by varying the cross section of the branches along its height.

A number of methods have been proposed for controlling the dynamics of thermoelectric cooling, in particular, using a time-variable current as the input of the thermoelement (for example, [1-3]). It should be noted that such a method has been found very effective for maintaining a prescribed temperature regime under the conditions of transient processes. However, in practice the use of such a method is not always possible because it requires a rather complex system for regulating the input current. A method for controlling a nonstationary temperature regime that is simpler to operate is one based on the use of thermoelements with branches having suitable nonprismatic shapes. It should be noted that for stationary conditions the change in geometric shape produces no effect, since the stationary limiting temperature drop is independent of the shape of the branches unless there is an additional heat input [4].

We consider the possibility of controlling a nonstationary cooling process by choosing the shape of the thermoelement, taking account of the fact that even with a constant current input the dynamic characteristics of the thermoelements will vary considerably.

We calculate the profile of the cross sections of a thermoelement possessing certain optimal dynamic characteristics. Suppose that each branch of the thermobattery is a rod with a cross section A(x) that varies with the height and having electric and thermal characteristics independent of coordinates and temperature. We consider the problem to be one-dimensional, neglecting the variation in temperature and current density over the cross section. Such a simplification is possible when the width of the thermoelement is small in comparison to its height. The temperature distribution is described by the heat-conduction equation

$$cA(x)\frac{\partial T}{\partial t} = \lambda\frac{\partial}{\partial x}\left(A(x)\frac{\partial T}{\partial x}\right) + \rho\frac{i^2}{A(x)} \quad (0 \leqslant x \leqslant d); \tag{1}$$

when x = 0,

$$\lambda\frac{\partial T}{\partial x} = e\frac{iT}{A_0} + g\frac{\partial T}{\partial t} - \alpha(T - T_0) - \frac{i^2\rho_C}{A_0}, \tag{2}$$

where $A(0) = A_0$;

$$T|_{x=d} = T|_{t=0} = T_0. \tag{3}$$

In a manner analogous to [5], we introduce dimensionless parameters: the Fourier and Biot criteria

$$\mathrm{Fo} = \frac{\lambda t}{cd^2}; \quad \mathrm{Bi} = \frac{\alpha d}{\lambda}; \quad \Theta = zT;$$

$$\nu_0 = \frac{edi}{\lambda A_0}; \quad \chi = \frac{x}{d}; \quad \eta = \frac{g}{cd}; \quad \xi = \frac{\rho_C}{\rho d}; \quad s = \frac{A(x)}{A_0}.$$

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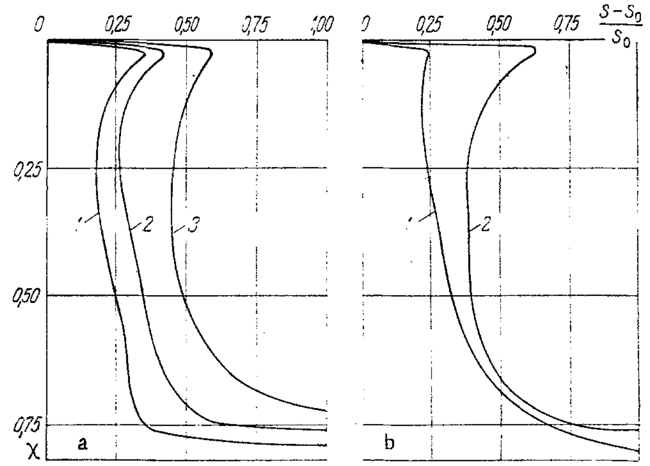

Fig. 1. Variation of the cross section with the height of the thermoelement: for Fig. 1a $\tilde{\Theta} = 0.47$; $\xi = 0.1$; 1) $\nu_0 = 0.483$; 2) 0.6; 3) 0.8; for Fig. 1b $\tilde{\Theta} = 0.47$; $\nu_0 = 0.6$; 1) $\xi = 0$; 2) 0.2.

Then Eq. (1) and the boundary conditions take the form

$$\frac{\partial \Theta}{\partial \mathrm{Fo}} - \frac{\partial^2 \Theta}{\partial \chi^2} = \frac{1}{s} \cdot \frac{\partial s}{\partial \chi} \cdot \frac{\partial \Theta}{\partial \chi} + \frac{\nu_0^2}{s^2} \quad (0 < \chi < 1,\ \mathrm{Fo} > 0), \tag{4}$$

$$\frac{\partial \Theta}{\partial \chi} = \nu_0 \Theta - \nu_0^2 \xi + \eta \frac{\partial \Theta}{\partial \mathrm{Fo}} + \mathrm{Bi}\,(\Theta - \Theta_0)|_{\chi=0}, \tag{5}$$

$$\Theta|_{\mathrm{Fo}=0} = \Theta|_{\chi=1} = \Theta_0,\ s(0) = 1. \tag{6}$$

We consider the problem of optimizing the shape of the branches. We must express the dimensionless area s as a function of χ in such a way that s(χ) and its derivative are continuous functions, where for a given length of time $\tilde{\mathrm{Fo}}$ the variation of the temperature at the cold junction $\Theta|_{\chi=0}$ will differ as little as possible from a given function $\Theta(\mathrm{Fo})$. As the criterion for the close approximation between $\Theta(\mathrm{Fo}, 0)$ and $\Theta(\mathrm{Fo})$ we select the functional

$$J = \int_0^{\tilde{\mathrm{Fo}}} [\Theta(\mathrm{Fo},\ 0) - \tilde{\Theta}(\mathrm{Fo})]^2 d\,\mathrm{Fo}. \tag{7}$$

This problem belongs to the class of problems concerned with the optimal control of systems whose behavior is described by partial differential equations [6]. In order to determine the necessary optimality conditions, we introduce the associated function v(Fo, χ) defined by the relations

$$\frac{\partial}{\partial x}\left(s^2 \frac{\partial v}{\partial \chi} + s \frac{\partial s}{\partial \chi} v\right) + s^2 \frac{\partial v}{\partial \mathrm{Fo}} = 2(\Theta|_{\chi=0} - \tilde{\Theta}(\mathrm{Fo})), \tag{8}$$

$$v\left(\nu_0 + \mathrm{Bi} - \frac{\partial s}{\partial \chi}\right) - \frac{\partial v}{\partial \chi} - \eta \frac{\partial v}{\partial \mathrm{Fo}} = 0|_{\chi=0}, \tag{9}$$

$$v|_{\mathrm{Fo}=\tilde{\mathrm{Fo}}} = v|_{\chi=1} = 0. \tag{10}$$

The equation for the conjugate function is constructed in a manner analogous to [7]; the Hamiltonian function will have the form

$$H = vf - f_0, \tag{11}$$

where f is the first part of Eq. (4); f_0 is the integrand of the functional (7).

From the condition that H is a maximum we find the necessary conditions for the optimal distribution of s(χ):

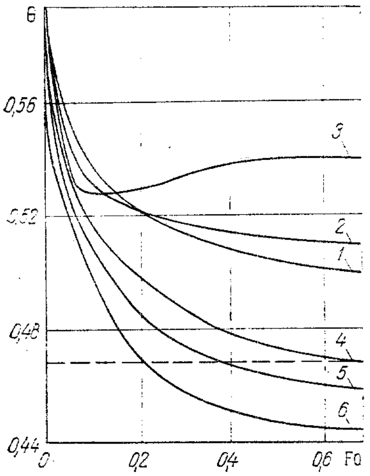

Fig. 2

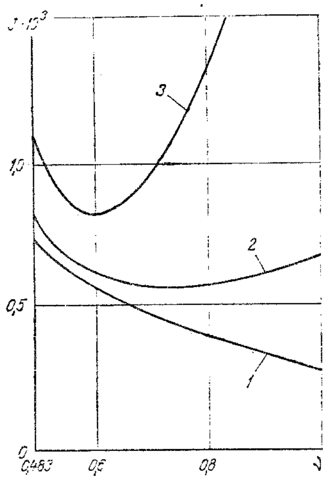

Fig. 3

Fig. 2. Variation of temperature with time for prismatic and optimal shapes: $\tilde{\Theta} = 0.47$; $\xi = 0.1$; 1) $\nu_0 = 0.483$; 2) 0.8; 3) 1; 4) 0.483; 5) 0.8; 6) 1.

Fig. 3. Minimized criterion as a function of current: $\tilde{\Theta} = 0.47$; 1) $\xi = 0$; 2) 0.1; 3) 0.3.

$$s(\chi) = \left(\frac{-2\nu_0^2 \int\limits_0^{\tilde{\mathrm{Fo}}} v \, d\mathrm{Fo}}{\frac{d}{d\chi} \int\limits_0^{\tilde{\mathrm{Fo}}} v \frac{\partial \Theta}{\partial \chi} \, d\mathrm{Fo}} \right)^{1/2}. \tag{12}$$

The desired distribution of $s(\chi)$ will be calculated by an iterative method. To do this, at each iteration $s^{(n)}(\chi)$ $(n = 0, 1, 2, \ldots)$ we calculate the corresponding temperature distribution $\Theta^{(n)}(\mathrm{Fo}, \chi)$ and the functions $v^{(n)}(\mathrm{Fo}, \chi)$, after which, by substituting their values into the right side of Eq. (12), we find the next approximation $s^{(n+1)}(\chi)$. As the zeroth approximation we take $s^{(0)}(\chi) = \mathrm{const} = 1$. The numerical integration of Eqs. (4)-(6) and (8)-(10) is carried out by a finite-difference method for the distribution of $s(\chi)$. We use a four-point difference scheme. The solution of the difference relations is obtained by the factorization method [8, 9].

We give some results of the calculation for the shape of the branches for the case in which the transient process at the operating junction of the thermoelement must approximate a stepwise temperature drop in a period of time $\tilde{\mathrm{Fo}} = 0.7$. The quantity $\Theta(\mathrm{Fo})$ was specified in the 0.45-0.483 range for $\Theta = 0.6$. The calculations were carried out for a wide range of values of ξ, η, and Bi, including zero values, and for a number of values of ν_0 beginning with 0.483 (the optimum value of the current density for a stationary regime in a thermoelement of prismatic shape). Taking account of the fact that the thermal diffusivity of effective thermoelectric materials is $6\text{-}8 \cdot 10^{-3}$ cm²/sec, we find that for d = 1 cm the given period of time corresponds to about 2 min.

Figure 1a shows examples of the optimal curves obtained for $s(\chi)$ for three current density values ν_0, other parameters being equal. It was found that the cross section of the thermoelement must increase sharply near the operating junction, within limits of 10% of the branch height. Then the cross section decreases and varies very little with height until we reach a value $\chi \cong 0.75$. The optimal value of s increases sharply near the hot junction. In the manufacture of thermoelements with variable branch cross section it must be taken into account that this segment has only a slight effect on the temperature at the operating junction for short cooling periods, and therefore when $\chi > 0.75$, we can take s = const. The graphs in Fig. 1a may be explained as follows. As the current ν_0 increases, there is an increase in the power of the Joule heat source at the junction (for $\chi = 0$ and $\xi \neq 0$) and in the volume of the element. Therefore, the corresponding optimal values of $s(\chi)$ will increase with increasing ν when $\chi > 0$, and this leads to reduced Joule heat generation at each cross section.

Figure 1b shows data characterizing the variation of the optimal shape when there is contact resistance. It follows from the figure that as the contact resistance ξ increases, we must increase the cross section of the element near the cold junction, thereby compensating for the increase in the power of the Joule heat source at the junction by a decrease in the volumetric heat generation near the junction.

The variation of temperature with time for various values of ν_0 is shown in Fig. 2 [curves 4, 5, and 6 are for optimal $s(\chi)$, and curves 1, 2, and 3 are for a thermoelement with prismatic branches]. The behavior of the curves shows that as the current density increases, so does the cooling rate. However, there is an optimal current density ($\nu_0 \cong 0.8$) for which the difference between the specified $\tilde{\Theta}(\mathrm{Fo}) = 0.47$ and the actual $\Theta(\mathrm{Fo})$ is minimal.

Figure 3 gives a family of curves showing how the mean-square deviation expressed by the functional J varies with the current density ν_0 for various values of contact resistance. From the data shown we can see that the value of the minimized functional first decreases with increasing ν_0 and then increases. This increase is due to the fact that for high current densities there is a considerable temperature drop, exceeding the required value even for small values of time. When there is no contact resistance ξ, the functional J monotonically decreases as the current increases (curve 1).

As a reference value, we select the value of the functional J_0 calculated for a thermoelement of prismatic shape [$s(\chi)$ = const]. From a comparison of J_0 with the values of J calculated for the optimal shape it can be seen that J can decrease to 10-20% of J_0. It should also be noted that because the cross section $s(\chi)$ varies with height in the nonstationary regime, when the current is constant with respect to time, the temperature drop, as can be seen from Fig. 2, may considerably exceed the limiting level attained in the stationary regime ($\Theta_{\min}^{\mathrm{stat}} = 0.483$ for $\Theta_0 = 0.6$).

In addition to being faster, this method enables us to calculate the shape of a thermoelement for reproducing a prescribed temperature variation. For example, it is possible to produce a thermoelement of such shape that for constant current the temperature will decrease almost linearly in a certain interval of time.

NOTATION

T, absolute temperature; t, time; x, length coordinate; d, height of thermoelement; i, current intensity; λ, c, ρ, thermal conductivity, volumetric heat capacity, and resistivity of the thermoelement branches, respectively; e, coefficient of thermo-emf of the thermocouple; $z = e^2/\rho\lambda$, thermoelectric quality factor; α, convective heat-transfer coefficient; g, heat capacity of the switching plate and the cooled object; ρ_c, contact resistance (the values of α, g, and ρ_c are those for a unit working surface).

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